SURFACE TENSION

THE FORCE OF COHESION

The force of attraction between the molecules of the same substance is called cohesion.

In case of solids, the force of cohesion is very large and due to this solids have definite shape and size. On the other hand, the force of cohesion in case of liquids is weaker than that of solids. Hence liquids do not have definite shape but have definite volume. The force of cohesion is negligible in case of gases. Because of this fact, gases have neither fixed shape nor volume.

Examples.

(i) Two drops of a liquid coalesce into one when brought in mutual contact because of the cohesive force.

(ii) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.

(iii) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

FORCE OF ADHESION

The force of attraction between molecules of different substances is called adhesion.

Examples.

(i) Adhesive force enables us to write on the black board with a chalk.

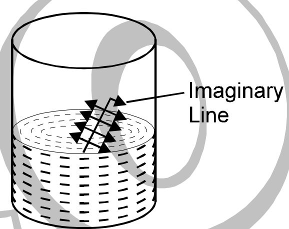
(ii) Adhesive force helps us to write on the paper with ink.

(iii) Large force of adhesion between cement and bricks helps us in construction work.

(iv) Due to force of adhesive, water wets the glass plate.

(v) Fevicol and gum are used in gluing two surfaces together because of adhesive force.

SURFACE TENSION



Surface Tension is a property of liquid at rest by virtue of which a liquid surface gets contracted to a minimum area and behaves like a stretched membrane.

Surface Tension of a liquid is measured by force per unit length on either side of any imaginary line drawn tangentially over the liquid surface, force being normal to the imaginary line as shown in the figure.

i.e. Surface tension

$$(T) = \frac{\text{Total force on either of the imginary line (F)}}{\text{Length of the line } (\ell)}$$

Units of Surface Tension.

In C.G.S. system the unit of surface tension is dyne/cm (dyne cm^{-1}) and SI system its units is Nm^{-1}

EXPLANATION OF SOME OBSERVED PHENOMENA

1. Lead balls are spherical in shape.

2. Rain drops and a globule of mercury placed on glass plate are spherical.

3. Hair of a shaving brush/painting brush, when dipped in water spread out, but as soon as it is taken out. Its hair stick together.

4. A greased needle placed gently on the free surface of water in a beaker does not sink.

5. Similarly, insects can walk on the free surface of water without drowning.

6. Bits of Camphor gum move irregularly when placed on water surface.

SURFACE ENERGY

We know that the molecules on the liquid surface experience net downward force. So to bring a molecule from the interior of the liquid to the free surface, some work is required to be done against the intermolecular force of attraction, which will be stored as potential energy of the molecule on the surface. The potential energy of surface molecules per unit area of the surface is called surface energy.

Unit of surface energy is erg cm^{-2} in C.G.S. system and Jm^{-2} in SI system. Dimensional formula of surface energy is $[ML^{0}T^{-2}]$ Surface energy depends on number of surfaces e.g. a liquid drop is having one liquid air surface while bubble is having two liquid air surface.

Ex. 1 A mercury drop of radius 1 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expanded if surface tension of mercury is 35×10^{-3} N/m.

Sol. If drop of radius R is sprayed into n droplets of equal radius r, then as a drop has only one surface, the initial surface area will be $4\pi R^2$ while final area is $n(4\pi r^2)$. So the increase in area

$$\Delta S = n(4\pi r^2) - 4\pi R^2$$

So energy expended in the process,

$$W = T\Delta S = 4\pi T\,[nr^2 - R^2] \qquad .... (1)$$

Now since the total volume of n droplets is the same as that of initial drop, i.e.,

$$\frac{4}{3}\pi R^3 = n[(4/3)\,\pi r^3] \quad \text{or} \quad r = R/n^{1/3} \qquad ....(2)$$

Putting the value of r from equation (2) in (1)

$$\mathbf{W = 4\pi R^2 T\,(n)^{1/3} - 1].}$$

Ex. 2 If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given be

$$\frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$$

where T is the surface tension of water and J is the mechanical equivalent of heat.

Sol. Let n be the number of little droplets.

Since volume will remain constant, hence volume of n little droplets = volume of single drop

$$\therefore \quad n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad \text{or} \quad nr^3 = R^3$$

Decrease in surface area = $n \times 4\pi r^2 - r\pi R^2$

$$\text{or} \quad \Delta A = 4\pi\,[nr^2 - R^2] = 4\pi\left[\frac{nr^3}{r} - R^2\right] = 4\pi\left[\frac{R^3}{r} - R^2\right] = 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Energy evolved $W = T \times$ decrease in surface area $= T \times 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$

Heat produced, $Q = \frac{W}{J} = \frac{4\pi T R^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$ But $Q = ms\,d\theta$

where m is the mass of big drop, s is the specific heat of water and $d\theta$ is the rise in temperature.

$$\therefore \quad \frac{4\pi T R^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right] = \text{volume of big drop} \times \text{density of water} \times \text{sp. heat of water} \times d\theta$$

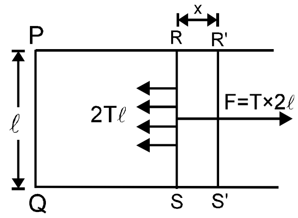
$$\text{or,} \quad \frac{4}{3}\pi R^3 \times 1 \times 1 \times d\theta = \frac{4\pi T R^3}{J}\left(\frac{1}{r} - \frac{1}{R}\right) \quad \text{or,} \quad d\theta = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

RELATION BETWEEN SURFACE TENSION AND SURFACE ENERGY

Consider a rectangular frame PQRS of wire, whose arm RS can slide on the arms PR and QS. If this frame is dipped in a soap solution, then a soap film is produced in the frame PQRS in fig. Due to surface tension (T), the film exerts a force on the frame (towards the interior of the film). Let ℓ be the length of the arm RS, then the force acting on the arm RS towards the film is. $F = T \times 2\ell$ [Since soap film has two surfaces, that is why the length is taken twice].

Let the arm RS be displaced to a new position R'S' through a distance x

$\therefore$ work done, $W = Fx = 2T\ell x$



Increase in potential energy of the soap film.

$= EA = 2E\ell x$ = work done in increasing the area (ΔW)

where E = surface energy of the soap film per unit area.

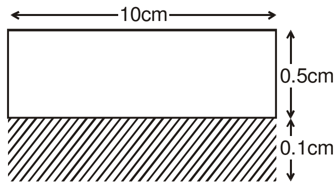
According the law of conservation of energy, the work done must be equal to the increase in the potential energy

$$\therefore\ 2T\ell x = 2E\ell x \text{ or } T = E = \frac{\Delta W}{A}$$

Thus, surface tension is numerically equal to surface energy or work done per unit increase surface area.

Ex. 3 A film of water is formed between two straight parallel wires each 10 cm long and at a separation 0.5 cm. Calculate the work required to increase 1 mm distance between them.
Surface tension of water = 72×10^{-3} N/m.

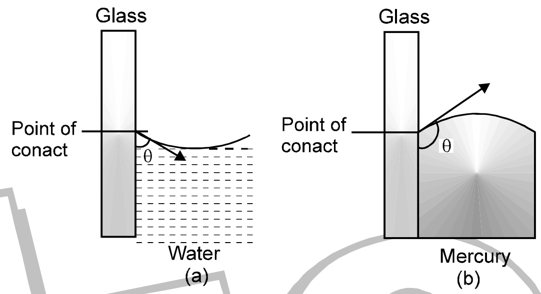
Sol. Here the increase in area is shown by shaded portion in the figure.
Since this is a water film, it has two surfaces, therefore
increase in area, $\Delta S = 2 \times 10 \times 0.1 = 2\ cm^2$



$\therefore$ Work required to be done,
$W = \Delta S \times T$
$= 2 \times 10^{-4} \times 72 \times 10^{-3}$
$= 144 \times 10^{-7}$ joule
$= 1.44 \times 10^{-5}$ joule

ANGLE OF CONTACT

The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called angle of contact. Those liquids which wet the walls of the container (say in case of water and glass) have meniscus concave upwards and their value of angle of contact is less than 90° (also called acute angle). However, those liquids which don't wet the walls of the container (say in case of mercury and glass) have meniscus convex upwards and their value of angle of contact is greater than 90° (also called obtuse angle). The angle of contact of mercury with glass about 140°, whereas the angle of contact of water with glass is about 8°. But, for pure water, the angle of contact θ with glass is taken as 0°.



Shape of Liquid Meniscus

When a capillary tube or a tube is dipped in a liquid, the liquid surface becomes curved near the point of contact. This curved surface is due to the two forces i.e.

(i) due to the force of cohesion and

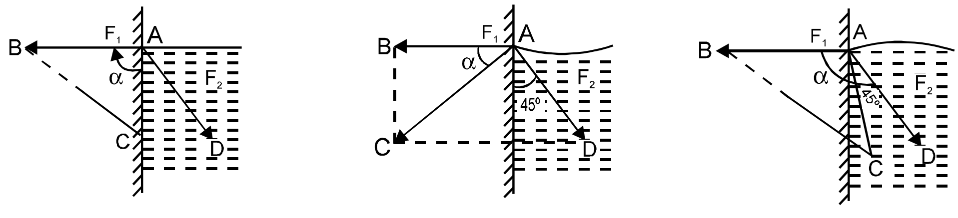
(ii) due to the force of adhesion. The curved surface of the liquid is called **meniscus of the liquid.**
Various forces acting on molecule A are:

(i) Force F_1 due to force of adhesion which acts outwards at right angle to the wall of the tube. This force is represented by AB.

(ii) Force F_2 due to force of cohesion which acts at an angle of 45° to the vertical. This force is represented by AD.

(iii) The weight of the molecule A which acts vertically downward along the wall of the tube.

Since the weight of the molecule is negligible as compared to F_1 and F_2 and hence can be neglected. Thus, there are only two forces (F_1 and F_2) acting on the liquid molecules. These forces are inclined at an angle of 135°.



(a) (b) (c)

The resultant force represented by AC will depend upon the values of F_1 and F_2. Let the resultant force makes an angle α with F_1.
According to parallelogram law of vectors.

$$\tan\alpha = \frac{F_2 \sin 135^\circ}{F_1 + F_2 \cos 135^\circ} = \frac{F_2/\sqrt{2}}{F_1 - F_2/\sqrt{2}} = \frac{F_2}{\sqrt{2}F_1 - F_2}$$

Special cases :

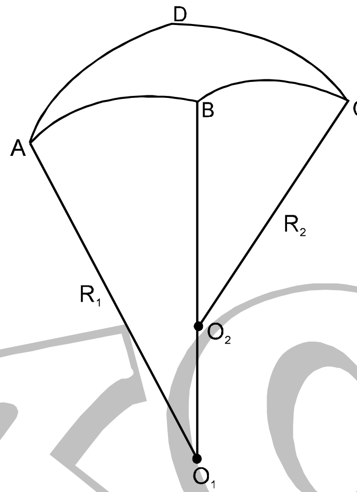
(i) If $F_2 = \sqrt{2}\, F_1$, then $\tan \alpha = \infty \therefore \alpha = 90°$
Then the resultant force will act vertically downward and hence the meniscus will be plane or horizontal shown in figure (a). Example; pure water contained in silver capillary tube.

(ii) If $F_2 < \sqrt{2}\, F_1$, then $\tan \alpha$ is positive $\therefore$ α is acute angle
Thus, the resultant will be directed outside the liquid and hence the meniscus will be concave upward shown in figure (b). This is possible in case of liquids which wet the walls of the capillary tube. Example ; water in glass capillary tube.

(iii) If $F_2 > \sqrt{2}\, F_1$, then $\tan \alpha$ is negative $\therefore$ α is obtuse angle.
Thus, the resultant will be directed inside the liquid and hence the meniscus will be convex upward shown in figure (c). This is possible in case of liquids which do not wet the walls of the capillary tube. Example ; mercury in glass capillary tube.

RELATION BETWEEN SURFACE TENSION, RADII OF CURVATURE AND EXCESS PRESSURE ON A CURVED SURFACE.

Let us consider a small element ABCD (fig.) of a curved liquid surface which is convex on the upper side. R_1 and R_2 are the maximum and minimum radii of curvature respectively, They are called the 'principal radii of curvature' of the surface. Let p be the excess pressure on the concave side.

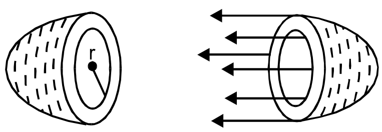


then $p = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$. If instead of a liquid surface, we have a liquid film, the above expression will be

$p = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, because a film has two surface.

EXCESS PRESSURE INSIDE A LIQUID DROP AND A BUBBLE

1. **Inside a bubble :** Consider a soap bubble of radius r. Let p be the pressure inside the bubble and p_a outside. The excess pressure = $p - p_a$. Imagine the bubble broken into two halves, and consider one half of it as shown in Fig. Since there are two surfaces, inner and outer, so the force due to surface tension is



F = surface tension x length = T x 2 (circumference of the bubble) = T x 2 (2 Tπ r) ... (1)
The excess pressure $(p - p_a)$ acts on a cross-sectional area πr^2, so the force due to excess pressure is $\Rightarrow$ $F = (p - p_a)\, \pi r^2$ .... (2)
The surface tension force given by equation (1) must balance the force due to excess pressure given by equation (2) to maintain the equilibrium. i.e. $(p - p_a)\, \pi r^2 = T \times 2\, (2\pi r)$

or $(p - p_a) = \frac{4T}{r} = p_{excess}$

above expression can also be obtained by equation of excess pressure of curve surface by putting $R_1 = R_2$.

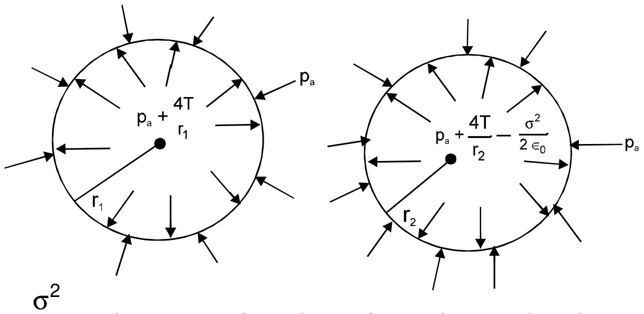
2. **Inside the drop :** In a drop, there is only one surface and hence excess pressure can be written as

$$(p - p_a) = \frac{2T}{r} = p_{excess}$$

3. **Inside air bubble in a liquid :** $(p - p_a) = \frac{2T}{r} = p_{excess}$

4. **A charged bubble :**
If bubble is charged, it's radius increases.
Bubble has pressure excess due to charge too.

Initially pressure inside the bubble = $p_a + \dfrac{4T}{r_1}$



For charge bubble, pressure inside = $p_a + \dfrac{4T}{r_2} - \dfrac{\sigma^2}{2\epsilon_0}$. where σ surface is surface charge density.
Taking temperature remains constant then from Boyle's law

$$\left(p_a + \frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^3 = \left[p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\epsilon_0}\right]\frac{4}{3}\pi r_2^3$$

From above expression the radius of charged drop may be calculated. It can conclude that radius of charged bubble increases, i.e. $r_2 > r_1$

Ex. 4 A minute spherical air bubble is rising slowly through a column of mercury contained in a deep jar. If the radius of the bubble at a depth of 100 cm is 0.1 mm, calculate its depth where its radius is 0.126 mm, given that the surface tension of mercury is 567 dyne/cm. Assume that the atmospheric pressure is 76 cm of mercury.

Sol. The total pressure inside the bubble at depth h_1 is (P is atmospheric pressure)

$$= (P + h_1 \rho g) + \frac{2T}{r_1} = P_1$$

and the total pressure inside the buffer at depth h_2 is = $(P + h_2 \rho g) + \dfrac{2T}{r_2} = P_2$

Now, according to Boyle's Law

$$P_1V_1 = P_2V_2 \quad \text{where} \quad V_1 = \frac{4}{3}\pi r_1^3, \quad \text{and} \quad V_2 = \frac{4}{3}\pi r_2^3$$

Hence we get
$$\left[(P + h_1 \rho g) + \frac{2T}{r_1}\right]\frac{4}{3}\pi r_1^3 = \left[(P + h_2 \rho g) + \frac{2T}{r_2}\right]\frac{4}{3}\pi r_2^3$$

or,
$$\left[(P + h_1 \rho g) + \frac{2T}{r_1}\right] r_1^3 = \left[(P + h_2 \rho g) + \frac{2T}{r_2}\right] r_2^3$$

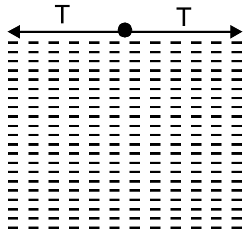
Given that : h_1 = 100 cm, r_1 = 0.1 mm = 0.01 cm, r_2 = 0.126 mm = 0.0126 cm, T = 567 dyne/cm, P = 76 cm of mercury. Substituting all the values, we get

h_2 = 9.48 cm.

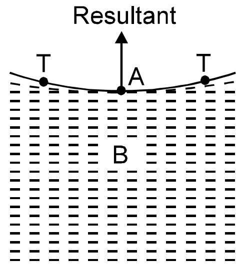
EXCESS OF PRESSURE INSIDE A CURVED SURFACE

1. **Plane Surface :** If the surface of the liquid is plane [as shown in Fig.(a)], the molecule on the liquid surface is attracted equally in all directions. The resultant force due to surface tension is zero. The pressure, therefore, on the liquid surface is normal.

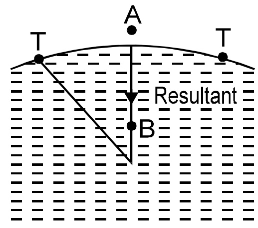
2. **Concave Surface :** If the surface is concave upwards [as shown in Fig.(b)], there will be upward resultant force due to surface tension acting on the molecule. Since the molecule on the surface is in equilibrium, there must be an excess of pressure on the concave side in the downward direction to balance the resultant force of surface tension $p_A - p_B = \dfrac{2T}{r}$.



(a) Plane Surface



(a) Concave Surface



(c) Convex Surface

3. **Convex Surface :** If the surface is convex [as shown in Fig.(c)], the resultant force due to surface tension acts in the downward direction. Since the molecule on the surface are in equilibrium, there must be an excess of pressure on the concave side of the surface acting in the upward direction to balance the downward resultant force of surface tension, Hence there is always an excess of pressure on concave side of a curved surface over that on the convex side. $p_B - p_A = \frac{2T}{r}$

Ex. 5 A barometer contains two uniform capillaries of radii 1.44×10^{-3} m and 7.2×10^{-4} m. If the height of the liquid in the narrow tube is 0.2 m more than that in the wide tube, calculate the true pressure difference. Density of liquid = 10^3 kg/m^3, surface tension = 72×10^{-3} N/m and g = 9.8 m/s^2.

Sol. Let the pressure in the wide and narrow capillaries of radii r_1 and r_2 respectively be P_1 and P_2.
Then pressure just below the mensiscus in the wide and narrow tubes respectively are

$\left(P_1 - \frac{2T}{r_1}\right)$ and $\left(P_2 - \frac{2T}{r_2}\right)$ [excess pressure = $\frac{2T}{r}$].

Difference in these pressures $= \left(P_1 - \frac{2T}{r_1}\right) - \left(P_2 - \frac{2T}{r_2}\right) = h\rho g$

$\therefore$ True pressure difference $= P_1 - P_2$

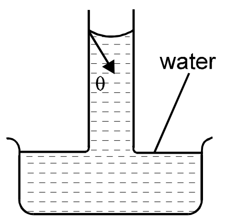
$$= h\rho g + 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= 0.2 \times 10^3 \times 9.8 + 2 \times 72 \times 10^{-3}\left[\frac{1}{1.44\times10^{-3}} - \frac{1}{7.2\times10^{-4}}\right]$$

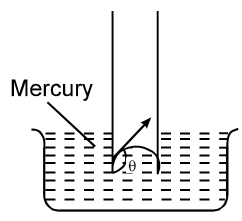
$= 1.86 \times 10^3 =$ **1860 N/m²**

CAPILLARITY

A glass tube of very fine bore throughout the length of the tube is called capillary tube. If the capillary tube is dipped in water, the water wets the inner side of the tube and rises in it [shown in figure (a)]. If the same capillary tube is dipped in the mercury, then the mercury is depressed [shown in figure (b)]. The phenomenon of rise or fall of liquids in a capillary tube is called capillarity.



(a)



(b)

PRACTICAL APPLICATIONS OF CAPILLARITY

1. The oil in a lamp rises in the wick by capillary action.

2. The tip of nib of a pen is split up, to make a narrow capillary so that the ink rises upto the tin or nib continuously.

3. Sap and water rise upto the top of the leaves of the tree by capillary action.

4. If one end of the towel dips into a bucket of water and the Other end hangs over the bucket the towel soon becomes wet throughout due to capillary action.

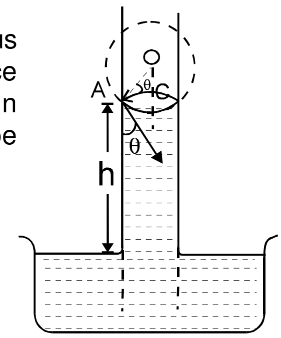
5. Ink is absorbed by the blotter due to capillary action.

6. Sandy soil is more dry than clay. It is because the capillaries between sand particles are not so fine as to draw the water up by capillaries.

7. The moisture rises in the capillaries of soil to the surface, where it evaporates. To preserve the moisture m the soil, capillaries must be. broken up. This is done by ploughing and leveling the fields

8. Bricks are porous and behave like capillaries.

CAPILLARY RISE (HEIGHT OF A LIQUID IN A CAPILLARY TUBE) ASCENT FORMULA



Consider the liquid which wets the walls of the tube, forms a concave meniscus shown in figure. Consider a capillary tube of radius r dipped in a liquid of surface tension T and density p. Let h be the height through which the liquid rises in the tube. Let p be the pressure on the concave side of the meniscus and p_a be the pressure on the convex side of the meniscus. The excess pressure

$(p - p_a)$ is given by $(p - p_a) = \frac{2T}{R}$

Where R is the radius of the meniscus. Due to this excess pressure, the liquid will rise in the capillary tube till it becomes equal to the hydrostatic pressure hpg. Thus in equilibrium state.

Excess pressure = Hydrostatic pressure or $\frac{2T}{R} = h\rho g$

Let θ be the angle of contact and r be the radius of the capillary tube shown in the fig.

From ΔOAC, $\frac{OC}{OA} = \cos\theta$ or $R = \frac{r}{\cos\theta}$ $\Rightarrow h = \frac{2T\cos\theta}{r\rho g}$

This expression is called **Ascent formula.**

Discussion.

(i) For liquids which wet the glass tube or capillary tube, angle of contact θ < 90°. Hence cos θ = positive. ⇒ h = positive. It means that these liquids rise in the capillary tube. Hence, **the liquids which wet capillary tube rise in the capillary tube**. For example, water, milk, kerosene oil, patrol etc.

Ex. 6 A liquid of specific gravity 1.5 is observed to rise 3.0 cm in a capillary tube of diameter 0.50 mm and the liquid wets the surface of the tube. Calculate the excess pressure inside a spherical bubble of 1.0 cm diameter blown from the same liquid.

Sol. The surface tension of the liquid is

$$T = \frac{rh\rho g}{2}$$

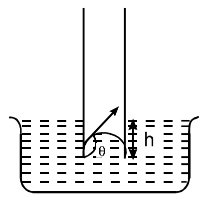
$$= \frac{(0.025\,cm)\,(3.0\,cm)\,(1.5\,gm/cm^3)\,(980\,cm/sec^2)}{2}$$

= 55 dyne/cm.

Hence excess pressure inside a spherical bubble

$$p = \frac{4T}{R} = \frac{4 \times 55\,dyne/cm}{(0.5\,cm)} = \mathbf{440\ dyne/cm^2}.$$

(ii) For liquids which do not wet the glass tube or capillary tube, angle of contact θ > 90°. Hence cos θ = negative ⇒ h = negative. Hence, the liquids which do not wet capillary tube are depressed in the capillary tube. For example, mercury.

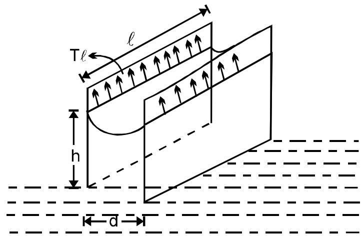


(iii) T, θ, ρ and g are constant and hence $h \propto \frac{1}{r}$. Thus, the liquid rises more in a narrow tube and less la a wider tube. This is called **Jurin's Law.**

(iv) If two parallel plates with the spacing 'd' are placed in water reservoir, then height of rise

$\Rightarrow \quad 2T\ell = \rho \ell h d g$

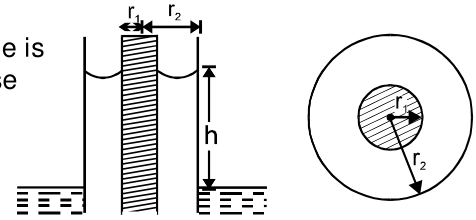
$$h = \frac{2T}{\rho d g}$$



(v) If two concentric tubes of radius 'r_1' and 'r_2' (inner one is solid) are placed in water reservoir, then height of rise

$\Rightarrow \quad T\,[2\pi r_1 + 2\pi r_2] = [\pi r_2^2\, h - \pi r_1^2 h]\,\rho g$

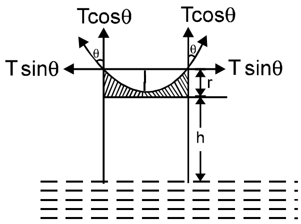
$$h = \frac{2T}{(r_2 - r_1)\rho g}$$



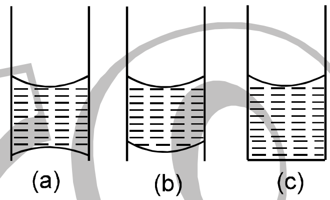
(vi) If weight of the liquid in the meniscus is to be consider :

$$T\cos\theta \times 2\pi r = [\pi r^2 h + \frac{1}{3}\,\pi r^2 \times r]\,\rho g$$

$$\left[h + \frac{r}{3}\right] = \frac{2T\cos\theta}{r\rho g}$$



(vii) When capillary tube (radius, 'r') is in vertical position, the upper meniscus is concave and pressure due to surface tension is directed vertically upward and is given by $p_1 = 2T/R_1$ where R_1 = radius of curvature of upper meniscus. When wetting is complete $p_1 = 2T/r$.



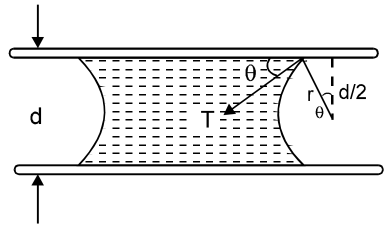
The hydrostatic pressure $p_2 = h\rho g$ is always directed downwards.
If $p_1 > p_2$ i.e. resulting pressure is directed upward. For equilibrium, the pressure due to lower meniscus should be downward. This makes lower meniscus concave downward (fig.a). The radius of lower meniscus R_2 can be given by $\frac{2T}{R_2} = (p_1 - p_2)$.

If $p_1 < p_2$, i.e. resulting pressure is directed downward for equilibrium, the pressure due to lower meniscus should be upward. This makes lower meniscus convex upward (fig. b).

The radius of lower meniscus can be given by $\frac{2T}{R_2} = p_2 - p_1$.

If $p_1 = p_2$, then is no resulting pressure. then, $p_1 - p_2 = \frac{2T}{R_2} = 0$ or, $R_2 = \infty$ i. e. lower surface will be FLAT. (fig.c).

(viii) **Liquid between two Plates -** When a small drop of water is placed between two glass plates put face to face, it forms a thin film which is concave outward along its boundary. Let 'R' and 'r' be the radii of curvature of the enclosed film in two perpendicular directions.



Hence the pressure inside the film is less than the atmospheric pressure outside it by an amount p given by $p = T\left(\frac{1}{r} + \frac{1}{R = \infty}\right)$ and we have. $p = \frac{T}{r}$.

If d be the distance between the two plates and θ the angle of contact for water and glass, then, from the figure, $\cos\theta = \frac{\frac{1}{2}d}{r}$ or $\frac{1}{r} = \frac{2\cos\theta}{d}$.

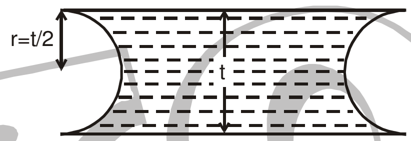
Substituting for $\frac{1}{r}$ in , we get $p = \frac{2T}{d}\cos\theta$.

θ can be taken zero for water and glass, i.e. $\cos\theta = 1$. Thus the upper plate is pressed downward by the atmospheric pressure minus $\frac{2T}{d}$. Hence the resultant downward pressure acting on the upper plate is $\frac{2T}{d}$. If A be the area of the plate wetted by the film, the resultant force F pressing the upper plate downward is given by F = resultant pressure × area $= \frac{2TA}{d}$.

For very nearly plane surface, d will be very small and hence the pressing force F very large. Therefore it will be difficult to separate the two plates normally.

Ex. 7 A drop of water volume 0.05 cm^3 is pressed between two glass-plates, as a consequence of which, it spreads and occupies an area of 40 cm^2. If the surface tension of water is 70 dyne/cm, find the normal force required to seperate out the two glass plates in newton.

Sol. Pressure inside the film is less than outside by an amount, $P = T\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$, where r_1 and r_2 are the radii of curvature of the meniscus. Here $r_1 = t/2$ and $r_2 = \infty$, then the force required to separate the two glass-plates, between which a liquid film is enclosed (figure) is, $F = P \times A = \frac{2AT}{t}$, where t is the thickness of the film, A = area of film.



$$F = \frac{2A^2T}{At} = \frac{2A^2T}{V} = \frac{2\times(40\times10^{-4})^2\times(70\times10^{-3})}{0.05\times10^{-6}}$$

$= 45$ N

Ex. 8 A glass plate of length 10 cm, breadth 1.54 cm and thickness 0.20 cm weighs 8.2 gm in air. It is held vertically with the long side horizontal and the lower half under water. Find the apparent weight of the plate. Surface tension of water = 73 dyne per cm, g = 980 cm/sec^2.

Sol. Volume of the portion of the plate immersed in water is

$10 \times \frac{1}{2}(1.54) \times 0.2 = 1.54\ cm^3$.

Therefore, if the density of water is taken as 1, then upthrust

= wt. of the water displaced

= 1.54 × 1 × 980 = 1509.2 dynes.

Now, the total length of the plate in contact with the water surface is 2(10 + 0.2) = 20.4 cm,

$\therefore$ downward pull upon the plate due to surface tension

= 20.4 × 73 = 1489.2 dynes

$\therefore$ resultant upthrust

= 1509.2 – 1489.2

= 20.0 dynes $= \frac{20}{980}$

= = 0.0204 gm-wt.

$\therefore$ apparent weight of the plate in water

= weight of the plate in air – resultant upthrust

= 8.2 – 0.0204 = 8.1796 gm **Ans.**

Ex. 9 A glass tube of circular cross-section is closed at one end. This end is weighted and the tube folates vertically in water, heavy end down. How far below the water surface is the end of the tube? Given : Outer radius of the tube 0.14 cm, mass of weighted tube 0.2 gm, surface tension of water 73 dyne/cm and g = 980cm/sec^2.

Sol. Let ℓ be the length of the tube inside water. The forces acting on the tube are :

(i) Upthrust of water acting upward

$= \pi r^2 \ell \times 1 \times 980 \quad = \frac{22}{7} \times (0.14)^2\, \ell \times 980 = 60.368\, \ell$ dyne.

(ii) Weight of the system acting downward

$= mg = 0.2 \times 980 = 196$ dyne.

(iii) Force of surface tension acting downward

$= 2\pi rT$

$= 2 \times \frac{22}{7} \times 0.14 \times 73 = 64.24$ dyne.

Since the tube is in equilibrium, the upward force is balanced by the downward forces. That is,

$60.368\ \ell = 196 + 64.24 = 260.24.$

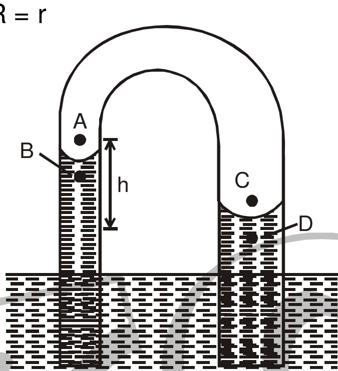
$\therefore \quad \ell = \frac{260.24}{60.368} = \mathbf{4.31\ cm.}$

Ex. 10 A glass U-tube is such that the diameter of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs? Surface tension of water is 0.07 nm^{-1}. Assume that the angle of contact between water and glass is 0º.

Sol. Suppose pressures at the points A, B, C and D be P_A, P_B, P_C and P_D respectively.
The pressure on the concave side of the liquid surface is greater than that on the other side by 2T/R.
An angle of contact θ is given to be 0º, hence R cos 0º = r or R = r

$\therefore \quad P_A = P_B + 2T/r_1$ and $P_C = P_D + 2T/r_2$

where r_1 and r_2 are the radii of the two limbs



But $P_A = P_C$

$\therefore \quad P_B + \frac{2T}{r_2} = P_D + \frac{2T}{r_2}$

or $\quad P_D - P_B = 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

where h is the difference in water levels in the two limbs

Now, $\quad h = \frac{2T}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Given that $\quad T = 0.07\ Nm^{-1}$, $\rho = 1000\ kgm^{-3}$

$r_1 = \frac{3}{2}$ mm $= \frac{3}{20}$ cm $= \frac{3}{20 \times 100}$ m $= 1.5 \times 10^{-3}$ m, $r_2 = 3 \times 10^{-3}$ m

$\therefore \quad h = \frac{2 \times 0.07}{1000 \times 9.8}\left(\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right)$ m $= 4.76 \times 10^{-3}$ m $= 4.76$ mm

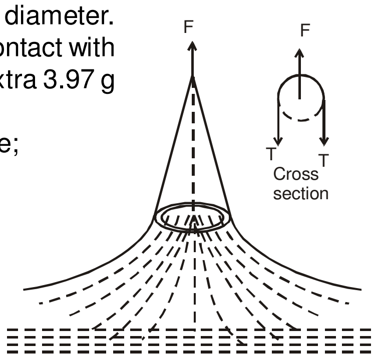
Ex. 11 A ring is cut form a platinum tube of 8.5 cm internal and 8.7 cm external diameter. It is supported horizontally from a pan of a balance so that it comes in contact with the water in a glass vessel. What is the surface tension of water if an extra 3.97 g weight is required to pull it away from water? (g = 980 cm/s^2).

Sol. The ring is in contact with water along its inner and outer circumference; so when pulled out the total force on it due to surface tension will be

$F = T\,(2\pi r_1 + 2\pi r_2)$

So, $\quad T = \frac{mg}{2\pi(r_1 + r_2)}$ $\quad [\because \ F = mg]$

i.e., $\quad T = \frac{3.97 \times 980}{3.14 \times (8.5 + 8.7)} = 72.13$ dyne/cm



Ex. 12 Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-shaped tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} Nm^{-1}. Take the angle of contact to be zero, and density of water to be 1.0×10^3 kg m^{-3} (g = 9.8 ms^{-2}).

Sol. Given that $r_1 = \frac{3.0}{2} = 1.5$ mm $= 1.5 \times 10^{-3}$ m, $r_2 = \frac{6.0}{2} = 3.0$ mm $= 3.0 \times 10^{-3}$ m,

$T = 7.3 \times 10^{-2}\ Nm^{-1}$, $\theta = 0^\circ$ $\rho = 1.0 \times 10^3\ kg\ m^{-1}$, $g = 9.8\ ms^{-2}$

When angle of contact is zero degree, the radius of the meniscus equals radius of bore.

Excess pressure in the first bore, $P_1 = \frac{2T}{r_2} = \frac{2 \times 7.3 \times 10^{-2}}{1.5 \times 10^{-3}}$ = 97.3 Pascal

Excess pressure in the second bore, $P_2 = \frac{2T}{r_2} = \frac{2 \times 7.3 \times 10^{-2}}{3 \times 10^{-3}}$ = 48.7 Pascal

Hence, pressure difference in the two limbs of the tube

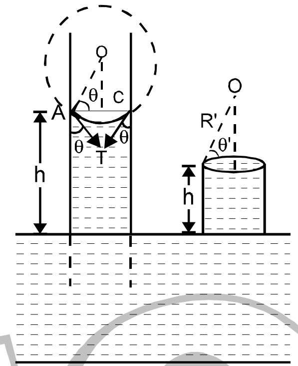
$$\Delta P = P_1 - P_2 = h\rho g$$

or $$h = \frac{P_1 - P_2}{\rho g} = \frac{97.3 - 48.7}{1.0 \times 10^3 \times 9.8} = 5.0 \text{ mm}$$

CAPILLARY RISE IN A TUBE OF INSUFFICIENT LENGTH

We know, the height through which a liquid rises in the capillary tube of radius r is given by

$$\therefore \quad h = \frac{2T}{R\rho g} \text{ or } hR = \frac{2T}{\rho g} = \text{constant}$$



When the capillary tube is cut and its length is less then h (i.e. h'), then the liquid rises upto the top of the tube and spreads in such a way that the radius (R') of the liquid meniscus increases and it becomes more flat so that hR = h'R' = Constant. Hence the liquid does not overflow.

If h' < h then R' > R or $\frac{r}{\cos\theta'} > \frac{r}{\cos\theta}$

$\Rightarrow \cos\theta' < \cos\theta$ $\Rightarrow$ $\theta' > \theta$

Ex. 13 If a 5 cm long capillary tube with 0.1 mm internal diameter open at both ends is slightly dipped in water having surface tension 75 dyne cm^{-1}, state whether (i) water will rise half way in the capillary. (ii) Water will rise up to the upper and of capillary (iii) Water will overflow out of the upper end of capillary/ Explain your answer.

Sol. Given that surface tension of water, T = 75 dyne/cm

Radius $r = \frac{0.1}{2}$ mm = 0.05 mm = 0.005 cm,

density ρ = 1 gm/cm^3, angle of contact, $\theta = 0^\circ$.

Let h be the height to which water rise in the capillary tube. Then

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 75 \times \cos 0^\circ}{0.005 \times 1 \times 981} \text{ cm} = \mathbf{30.58\ cm}.$$

But length of capillary tube, h' = 5 cm

(i) Because $h > \frac{h'}{2}$ therefore the first possibility does not exist.

(ii) Because the tube is of insufficient length therefore the water will rise upto the upper end of the tube.

(iii) The water will not overflow out of the upper end of the capillary. It will rise only upto the upper end of the capillary.

The liquid meniscus will adjust its radius of curvature R' in such a way that

$$R'h' = Rh \qquad \left[\because \quad hR = \frac{2T}{\rho g} = \text{constant}\right]$$

where R is the radius of curvature that the liquid meniscus would possess if the capillary tube were of sufficient length

$$\therefore \quad R' = \frac{Rh}{h} = \frac{rh}{h'} \left[\because \quad R = \frac{r}{\cos\theta} = \frac{r}{\cos 0^\circ} = r\right] = \frac{0.005 \times 30.58}{5} = \mathbf{0.0306\ cm}$$

APPLICATIONS OF SURFACE TENSION

(i) The wetting property is made use of in detergents and waterproofing. When the detergent materials are added to liquids, the angle of contact decreases and hence the wettability increases. On the other hand, when water proofing material is added to a fabric, it increases the angle of contact, making the fabric water-repellant.

(ii) The antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension the antiseptics spreads properly over the wound. The lubricating oils and paints also have low surface tension. So they can spread properly.

(iii) Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.

(iv) Oil spreads over the surface of water because the surface tension of oil is less than the surface tension of cold water.

(v) A rough sea can be calmed by pouring oil on its surface.

EFFECT OF TEMPERATURE AND IMPURITIES ON SURFACE TENSION

The surface tension of a liquid decreases with the rise in temperature and vice versa. According to Ferguson, $T = T_0 \left(1 - \frac{\theta}{\theta_c}\right)^n$ where T_0 is surface tension at 0ºC, θ is absolute temperature of the liquid, θ_c is the critical temperature and n is a constant varies slightly from liquid and has mean value 1.21. This formula shows that the surface tension becomes zero at the critical temperature, where the interface between the liquid and its vapour disappears. It is for this reason that hot soup tastes better while machinery parts get jammed in winter.

The surface tension of a liquid changes appreciably with addition of impurities. For example, surface tension of water increases with addition of highly soluble substances like NaCl, $ZnSO_4$ etc. On the other hand surface tension of water gets reduced with addition of sparingly soluble substances like phenol, soap etc.

